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NASA Contractor Report 187452
ICASE Report No. 90–65

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Contract No. NAS1–18605
September 1990

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Operated by the Universities Space Research Association

NASA
National Aeronautics and
Space Administration

Langley Research Center
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DISSIPATIVE CONTROLLER DESIGNS FOR SECOND-ORDER DYNAMIC SYSTEMS

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ABSTRACT

The passivity theorem may be used to design robust controllers for structures with positive transfer functions. This paper extends this result to more general configurations using dissipative system theory. A stability theorem for robust, model-independent controllers of structures which lack collocated rate sensors and actuators is given. The theory is illustrated for non-square systems and systems with displacement sensors.

[1]This research was supported by the National Aeronautics and Space Administration under NASA Contract No. NAS1-18605. The author is currently with the Department of Electrical Engineering at the University of Waterloo, Waterloo, Ontario, N2L 3G1, CANADA.

1 Introduction

In this paper we are concerned with control of systems with second-order dynamics modelled by the following system of ordinary differential equations.

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Fu(t) \tag{1}$$

$$y(t) = C_d x(t) + C_v \dot{x}(t) + C_a \ddot{x}(t) \tag{1b}$$

For structures, this is an approximation to an (infinite-dimensional) partial differential equation model and this system of equations may be of very high order, although the structural matrices M, D and K are positive definite and typically sparse and symmetric.

Control theory for time-invariant linear systems which are described by first-order dynamic equations has been well established for decades, and many control software tools are available today for systems written in first-order forms. For applications, engineers can simply convert whatever models they have to the first-order forms and then use the existing tools to design the controllers.

For second-order systems, transforming to first-order form not only increases the dimension of the problem, but also destroys the sparsity of the structural matrices. Not only physical insight, but computational efficiency is often lost in conversion to first-order form. Existing control analysis and design software may not be able to handle such a large system. For example, solving a 1000-by-1000 Riccati equation is practically impossible with today's numerical techniques.

There are basically two ways to address the controller design problem for a large scale system. One way is to minimize the dimension of the system model, through some model reduction technique. The reduced model is used in the controller design and some robust design methodology is used so that the controller stabilizes the original high order model. This is the approach behind most H_∞ design for structures [6].

Another way is to design a controller which is independent of uncertainties in the system model. The advantage to this approach is that, unlike the first approach, the accuracy of the original high-order model (1) is not assumed, nor is an accurate system identification required. This is very appealing, since at the current time, no accurate model of structural damping exists, and the existing models of stiffness are only accurate at low frequencies. Furthermore, damping is difficult to identify experimentally.

Structures with collocated rate sensors and force actuators ($C_v' = F$) are passive, *i.e.* have positive transfer functions [2]. The passivity theorem [1, 3] implies that any positive controller will lead to a stable closed loop system. This fact can be used to design control systems which remain stable despite large modelling errors. Recently, several researchers

have incorporated this approach with other robustness theorems in robust controller design for systems which have collocated rate sensors and force actuators but whose transfer functions may be non-positive due to computation delays, actuator dynamics *etc.* [7]. However, the restriction of collocated sensors and actuators is a stringent one, and excludes many applications. In particular, systems with displacement sensors and systems with a different number of inputs and outputs cannot be handled with this approach.

Since altering the configuration of sensors and actuators does not alter the fact that structures with positive damping dissipate energy, the question arises as to whether the passivity theorem can be generalized. For instance, when a mass-spring-dashpot is attached to any mechanical system, including flexible space structures, the damping of the system is almost always augmented regardless of the system size. The parameters of the mass-spring-dashpot are arbitrary, model-independent and thus insensitive to the system uncertainties. Knowledge of the system model may be used in adjusting the parameters in order to satisfy system performance requirements. However, changes or errors in the system will not destabilize the system because it is an energy-dissipative device. The question arises as to if there are any feedback controller designs using sensors and actuators which behave like the mass-spring-dashpot device. The answer is positive as shown in [4], wherein a conventional engineering approach was used to design active controllers which behave like the mass-spring-dashpot device. Although the conventional approach provides physical insights, it lacks the theoretical basis necessary for application to more complicated systems such as nonlinear flexible robots. This paper is motivated by the desire to provide a rigorous theory to support this argument based on physical intuition.

In the next section we summarize results on *dissipative system theory* obtained mainly by Willems [8, 9]. A stability theorem which generalizes the passivity and small gain theorems is given. In order to facilitate the application of these ideas to non-linear models of structural dynamics, no assumption of linearity is made. In subsequent sections we apply this theory to derive conditions for robust, model insensitive control of (1) non-square systems with rate sensors (2) systems with displacement sensors. As an example, we present a rigorous proof of the stability of a mass-spring-dashpot type controller.

2 Dissipative System Theory

Intuitively, a dissipative system is one which dissipates energy. In other words, the energy stored in such a system will be less than the energy supplied to it. This concept can be written formally in terms of a *storage function* which generalizes the concept of energy. The theory in this section is taken from [5, 8, 9].

Definition 2.1: A dynamical system Σ is defined as follows:

(i) $\mathcal{U}$ is the *input space* and consists of a class of U-valued functions on the positive real line.

(ii) $\mathcal{Y}$ is the *output space* and consists of a class of Y-valued functions on the positive real line.

(iii) Both Y and U are finite-dimensional inner product spaces and both $\mathcal{Y}$ and $\mathcal{U}$ are closed under the shift operator *i.e.* if $u(t) \in \mathcal{U}$ then $u(t+T) \in \mathcal{U}$.

(iv) Define $R_2^+ := \{(t_2, t_1) \in R \times R; t_2 \geq t_1\}$. The state transition function $\phi : R_2^+ \times X \times \mathcal{U} \mapsto X$ and defines the state through the relation $x(t_2) = \phi(t_2, t_1, x(t_1), u)$ This function satisfies the usual axioms for autonomous dynamical systems:

(a) Consistency of initial condition: $\phi(t, t, x_0, u) = x_0$

(b) Semigroup property: $\phi(t_2, t_1, \phi(t_1, t_0, x_0, u), u) = \phi(t_2, t_0, x_0, u)$

(c) Causality: $u_1(t) = u_2(t)$ *for* $t_0 < t < t_1$ *implies* $\phi(t, t_0, x_0, u_1) = \phi(t, t_0, x_0, u_2)$ for $t_0 < t < t_1$.

(d) Time Invariance: $\phi(t_1 + T, t_2 + T, x_0, u_1) = \phi(t_1, t_2, x_0, u_2)$ for all $T \geq 0, t_2 \geq t_1, u_2(t) = u_1(t+T)$.

(v) The function $r : X \times U \mapsto Y$ is the output function: $y(t) = r(x(t), u(t))$.

In view of the system time-invariance (4d) we will henceforth use $t_0 = 0$.

Definition 2.2: A dynamical system is said to be *dissipative* if there exists a nonnegative function $S : X \mapsto R^+$, called the *storage function* and a supply rate $w : U \times Y \mapsto R$ such that , for all $t_1 \geq 0$,

$$S(x_0) + \int_0^{t_1} w(u, y)dt \geq S(x_1) \tag{2}$$

where $u \in \mathcal{U}, x_1 = \phi(t_1, 0, x_0, u)$ and $y = r(x, u)$.

Note that the storage function is, in general, not unique. The following theorem shows that if an appropriate supply function is found, it is not necessary to actually define a storage function.

Theorem 2.1 *[8] If there exists a supply function $w(u, y)$ for Σ such that*

$$\int_0^{t_1} w(u, y)dt \geq 0$$

for all $t_1 \geq 0$ then there exists a storage function S such that (2) is satisfied and so Σ is a dissipative system.

Moylan and Hill [5] define a useful class of dissipative systems .

Definition 2.3: [5] Given P, Q, R of appropriate dimension with Q, R symmetric, define the supply rate

$$w(u,y) = (y, Py) + (y, Qu) + (u, Ru) \tag{3}$$

A system is (P, Q, R)-dissipative if

$$\int_0^{t_1} w(u,y) \geq 0 \qquad \forall t_1 \geq 0.$$

Passivity, or positivity, can now be seen to be the special case where $P = R = 0$ and $Q = I$. Strict passivity is obtained by defining $R = -\delta^2 I$ and $Q = I$. Similiarly, we obtain input-output stable systems by choosing $P = -I$ and $R = k^2 I$ where k is the gain.

Suppose G is a given system, for which we wish to design a controller H, arranged as shown in Figure 1.

The feedback system, or alternatively the pair (G, H), is said to be *externally stable* if $u_1, u_2 \in L_2(0, \infty, U)$ implys $y_1, y_2 \in L_2(0, \infty, Y)$, and there is a maximum ratio, the L_2 gain, between the norm of the input and the norm of the output.

The following theorem provides a simple test for external stability of interconnected (P, Q, R)-dissipative systems .

Theorem 2.2 *[5][Theorem 1] Consider systems G and H connected in the familar feedback configuration shown in Figure 1, and assume that H is (P_2, Q_2, R_2)-dissipative and that G is (P_1, Q_1, R_1)-dissipative. The closed loop system is externally stable if*

$$\hat{Q} := \begin{bmatrix} R_1 + P_2 & Q_1' - Q_2 \\ Q_2' - Q_1 & R_2 + P_1 \end{bmatrix} \tag{4}$$

is negative-definite. □

The well-known passivity and small-gain theorems [3] can easily be derived as special cases of Theorem 2.2.

In order to study stability of feedback systems with more general supply rates, we state an observability assumption.

Assumption 1: There exists some $T > 0$ and a non-negative continuous function $\alpha : R \mapsto R$, with $\alpha(0) = 0$ and $\alpha(\sigma) > 0$ for $\sigma > 0$, such that for identically zero input and any initial state x_0 , we have

$$\int_0^T y'(t)y(t)dt \geq \alpha(\| x_0 \|). \ \square$$

(For finite-dimensional linear time-invariant systems, this is equivalent to the standard definition of observability.)

Theorem 2.3 *Suppose that*

(a) A given system G has a supply rate $w_1 := (y, P_1 y) + w_c(u,y) + (u, R_1 u)$ with

$$\int_0^{t_1} w_1(u,y)dt \geq 0,$$

(b) H is a system with supply rate $w_2 := (y, P_2 y) + w_c(y,u) + (u, R_2 u)$ and

$$\int_0^{t_1} w_2(u,y)dt \geq 0.$$

If $R_2 + P_1$ and $P_2 + R_1$ are negative definite , and both G and H satisfy Assumption 1, then the origin is an asymptotically stable equilibrium point of the closed loop system (Figure 1) with zero external inputs.

Proof: We will demonstrate a Lyapunov function for the closed loop system. Let S_1 and S_2 be storage functions for G and H respectively. Define $\underline{r} = (r_1, r_2)$ and similarly $\underline{y}$. We have $u_1 := r_1 - y_2$ and $u_2 := r_2 + y_1$. It is clear that the interconnected system (Figure 1) is dissipative with storage function $S := S_1 + S_2$ and supply rate

$$\bar{w}(\underline{r}, \underline{y}) = w_1(u_1, y_1) + w_2(u_2, y_2).$$

We will show that S is in fact a Lyapunov function for the closed loop system.

Since

$$S(\underline{x}_0) + \int_0^{t_1} \bar{w}(\underline{r}, \underline{y})dt \geq S(\underline{x}_1) \tag{5}$$

it follows that for $r_1 = r_2 = 0$,

$$\begin{aligned}
\dot{S} &\leq w_1(u_1, y_1) + w_2(u_2, y_2) \\
&= w_1(-y_2, y_1) + w_2(y_1, y_2) \\
&= y_2'(R_1 + P_2)y_2 \ + \ y_1'(P_1 + R_2)y_1 \\
&\leq 0
\end{aligned}$$

since the terms arising from the cross rate w_c cancel. Thus , $\dot{S}$ is negative definite (Assumption 1).

It now remains only to show that S is a positive definite function.

$$S(\underline{x}_0) \geq S(x_1) - \int_0^{t_1} \bar{w}(\underline{r}, \underline{y})dt \geq \int_0^{t_1} -y_2'(R_1 + P_2)y_2 dt + \int_0^{t_1} -y_1'(P_1 + R_2)y_1 dt.$$

Since both plant and controller satisfy Assumption 1, there exists a positive definite function β with

$$S(\underline{x}_0) \geq \beta(\| \underline{x}_0 \|)$$

and so S is a Lyapunov function for the closed loop system. The result follows. □

Theorems 2.2 and 2.3 will be used to derive controllers for uncertain structures.

3 Rate Sensors

Suppose we have a structural control system (1) with only rate sensors *i.e.* $C_d = C_a = 0$. It is known that if $C_v' = F$ then the system transfer function is positive, or equivalently, the system is $(0, I, 0)$-dissipative [1]. The passivity theorem can be used. In particular, any strictly positive stable controller will lead to a stable closed loop system. It is not necessary to determine the model (1) beyond ensuring that $C_v' = F$ and that the dynamics are second order.

In this section we extend this result to more general configurations.

Theorem 3.1 *Consider (1) with $C_d = C_a = 0$ and suppose that there exists an operator $Q : U \mapsto Y$ such that*

$$C_v' Q = F. \tag{6}$$

Then, for some $p^2 > 0$, the system (1) is $(-p^2 I, Q, 0)-$dissipative.

Proof: For arbitrary $p > 0$, define the supply rate

$$w(u,y) := (y, -p^2 y) + (y, Qu).$$

Define

$$u_T(t) := P_T u(t) := \begin{cases} u(t), & t \leq T \\ 0, & t > T \end{cases} \tag{7}$$

and let $y_T(t)$ be the output which corresponds to the input $u_T(t)$.

$$\begin{aligned}
\int_0^T w(u,y)dt &= \int_0^T -p^2(y_T, y_T) + (y_T, Qu_T)dt \\
&= \int_0^\infty -p^2(y_T, y_T) + (y_T, Qu_T)dt + \int_T^\infty p^2(y_T, y_T)dt \\
&\geq \int_0^\infty -p^2(y_T, y_T) + (y_T, Qu_T)dt \\
&= \frac{1}{2\pi}\int_{-\infty}^\infty -p^2(\hat{y}_T, \hat{y}_T) + (\hat{y}_T, Q\hat{u}_T)d\omega
\end{aligned}$$

where $\hat{y}$ denotes the Laplace transform of y.

Defining $v(\jmath\omega) := \jmath\omega(K - M\omega^2 + D\jmath\omega)^{-1}F\hat{u}_T(\jmath\omega)$, so $\hat{y} = C_v v$,

$$\begin{aligned}
\int_0^T w_G(u,y)dt &\geq \frac{1}{2\pi}\int_{-\infty}^\infty -p^2(C_v v, C_v v) + (v, F\hat{u}_T)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^\infty v(\jmath\omega)^*[-p^2 C_v{}' C_v + \frac{1}{\jmath\omega}(K - M\omega^2 + D\jmath\omega)]v(\jmath\omega)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^\infty v(\jmath\omega)^*[-p^2 C_d{}' C_d + D)]v(\jmath\omega)d\omega.
\end{aligned}$$

If we choose p^2 small enough so that $p^2 C_v' C_v < D$, the expression in square brackets is positive definite for all ω. $\square$

The following stability theorem is now immediate.

Theorem 3.2 *Assume we have a second order system with rate sensors, and $C_v'Q = F$, as in the statement of the previous theorem. If a given controller H is (1) externally L_2-stable and (2) QH is a positive transfer function, then the closed loop system is stable.*

Proof: The system is $(-p^2 I, Q, 0)$-dissipative for some $p > 0$ (Theorem 3.1). Let k be the controller gain and choose a, b so $k \leq b^2/a^2$ and $b^2 < p^2$. We have that the controller is $(-a^2 I, Q', b^2 I)$-dissipative. The result now follows from Theorem 2.2. $\square$

This result is more general in several ways than previous results depending on positivity of a structure. The plant is not constrained to be square and the operator Q is not required to bear any relation to a Lyapunov function for the system. However, the rate sensors and force actuators do need to be sufficiently "collocated" so that $C_v'Q = F$ for some Q.

4 Structures with Displacement Sensors

In many applications, sensing is performed with displacement sensors. If the control is still input as a force, then the system transfer function is by no means positive, even the extended sense of $(0, Q, 0)$- dissipativeness discussed in the previous section. We would like to obtain a robustness result similar to that obtained in the previous section. One approach to this problem is to use multipliers (Figure 2). Figure 2 can be shown to be equivalent to Figure 1. The multiplier function L is chosen so that a standard result such as the passivity theorem may be used for the transformed systems. Unfortunately, for a given plant, there is no guarantee that a suitable multiplier function exists, nor any clue of how to choose the multiplier function. Furthermore difficulties associated with ensuring causality of the transformed systems may arise. Details can be found in [3].

Looking at the situation from a physical viewpoint, replacing rate sensors on a structure with displacement sensors does not change the internal dynamics of the structure. Intuition tells us that the system should remain dissipative. If suitable supply and storage functions do exist, so that the system (1) with displacement sensors is dissipative, Theorem 2.3 can then be applied to design robust controllers.

In order to keep the discussion simple, we consider the situation where only measurements of displacement are made *i.e.* $C_v = C_a = 0$. We note first that the definition of supply functions can be extended to include functions which involve derivatives of the

inputs and outputs. The input and output spaces have to be defined appropriately so that the supply function is well-defined.

Theorem 4.1 *Consider the second-order system (1) with $C_a = C_v = 0$, assume that there exists Q so $C_d'Q = F$. Let $c^2 > 0$ be a lower bound on system damping so that $c^2 M < D$. Define the cross rate*

$$w_c(u,y) := c^2(y, Qu) + (\dot{y}, Qu). \tag{8}$$

Then there is a number $p^2 > 0$ so that this system is dissipative with respect to the supply rate

$$w_G(u,y) = -p^2(y,y) + w_c(u,y).$$

Proof: First note that $\dot{y}$ exists for all outputs y so that w_G is a well-defined supply function. As in Theorem 3.1 and in [3] we will show that

$$\int_0^T w_G(u,y)dt \geq 0$$

for arbitrary $T \geq 0$ by showing that its transfer function is positive. Dissipativeness of the system with this supply rate will then follow from Theorem 2.1.

Defining $u_T(t) := P_T u(t)$ as in equation (7), let $y_T(t)$ be the output which corresponds to the input $u_T(t)$.

$$\begin{aligned}
\int_0^T w_G(u,y)dt &= \int_0^T -p^2(y_T, y_T) + c^2(y_T, Qu_T) + (\dot{y_T}, Qu_T)dt \\
&= \int_0^\infty -p^2(y_T, y_T) + c^2(y_T, Qu_T) + (\dot{y_T}, Qu_T)dt + \int_T^\infty p^2(y_T, y_T)dt \\
&\geq \int_0^\infty -p^2(y_T, y_T) + c^2(y_T, Qu_T) + (\dot{y_T}, Qu_T)dt \\
&= \frac{1}{2\pi}\int_{-\infty}^\infty -p^2(\hat{y_T}, \hat{y_T}) + c^2(\hat{y_T}, Q\hat{u_T}) + (s\hat{y_T}, Q\hat{u_T})d\omega.
\end{aligned}$$

Defining $v(\jmath\omega) := (K - M\omega^2 + D\jmath\omega)^{-1}F\hat{u_T}(\jmath\omega)$,

$$\int_0^T w_G(u,y)dt \geq \frac{1}{2\pi}\int_{-\infty}^\infty v(\jmath\omega)^*[-p^2 C_d' C_d + c^2(K - M\omega^2) + D\omega^2]v(\jmath\omega)d\omega.$$

If we choose p^2 so that $p^2 C_d' C_d < c^2 K$, the expression in square brackets is positive definite for all ω and for arbitrary T,

$$\int_0^T w_G(u,y)dt \ \geq \ 0$$

as required. □

Theorem 4.2 *Consider the structural system defined in the previous theorem. Assume observability. The closed loop system (Figure 1) will be stable if a linear controller is (a) stable (b) dissipative with respect to the supply rate*

$$w(u,y) = w_c(y,u) = c^2(u,Qy) + (\dot{u},Qy) \tag{9}$$

where Q, c^2 and w_c are as defined in the statement of the previous theorem. The input space $\mathcal{U}$ is restricted to inputs with $\dot{u} \in \mathcal{L}_2(U)$.

Proof: Define the positive number p^2 as in the previous theorem. Since the controller is stable, it is $(-a^2I, 0, b^2I)$-dissipative where we choose $b^2 < p^2$ and b^2/a^2 large than the controller gain. Choose an observable realization for the controller. If condition (b) also holds, then it follows that the origin is a stable equilibrium point of the closed loop system with $u_1 = u_2 = 0$(Theorem 2.3). Since both plant and controller are linear and finite dimensional, it follows that the closed loop system is also externally stable. □

Errors in the model of the system stiffness K or actual damping larger than the estimate will not destabilize the closed loop system. Other than ensuring $C_d'Q = F$, the only knowledge of the structure required in designing a stabilizing controller is a lower bound on the damping so that the controller satisfies (9). If the actual system damping is greater than the estimate, the closed loop system will still be stable.

Note that this result could also have been obtained through defining the multiplier function $L = Q(c^2+s)$. We feel however that there are several advantages to using dissipative system theory. The first one is that supply rates such as $(u,\dot{y})$ may be defined without the problems of causality and stability associated with the multiplier function s. More importantly, we think that the the choice of supply rate is clearer when it is known that the system "dissipates energy" in some sense.

5 Example: Second Order Controllers

In order to illustrate the ideas discussed above, consider the second order dynamic system with displacement sensors and $C_d'Q = F$ for some Q as discussed in the previous section.

Suppose we wish to control this system with a combination of direct feedback and the displacements of a second order system. Mathematically, the proposed controller has the form

$$M_2\ddot{x}_2(t) + D_2\dot{x}_2(t) + K_2x(t) = F_2u_2(t) \tag{10}$$

$$y_2(t) = C_{d2}x_2(t) + Eu_2(t).$$

Since both plant and controller are stable some readers might expect stability of the closed loop system to follow if both plant and controller have positive damping. This is not true, as the following simple example illustrates.

Example : $\ddot{x}(t) + 8\dot{x}(t) + 100x(t) = u_1(t) \qquad \ddot{x}_2(t) + .001\dot{x}_2(t) + .1x_2(t) = u_2(t)$

$$u_1 = r_1 - x_2 \qquad u_2 = r_2 + x$$

While for damping $d = 8$, the closed loop is stable, *increasing* the plant damping to 12 will lead to an unstable closed loop.

However, by using the results from the previous section we can obtain a controller (10) which is unconditionally stable.

Theorem 5.1 *Suppose we have a observable second order plant (1) with* $C_v = C_a = 0$ *and (2) there exists* Q *such that* $C_d'Q = F$ *. If the controller (10) is implemented so that* $C_{d2}'Q' = -F_2$ *and the bias* E *satisfies* $QE \geq F_2{}'X'F_2$ *where* $X > K_2{}^{-1}$*, then the closed loop system is stable.*

Proof: From Theorems 4.1 and 4.2 the result will follow if we can show that the given system is dissipative with respect to the supply rate (9) for arbitrarily small $c^2 > 0$.

Define $v(j\omega) := (-M_2\omega^2 + D_2 j\omega + K_2)^{-1}\hat{u}_T(j\omega)$, and set $E'Q' = F_2'XF_2 + E_2$ where X, E_2 are as yet undetermined. Using the same frequency domain technique as in Theorems 3.1 and 4.1, we obtain

$$\int_0^T w_c(u,y)dt \geq c^2 \int_{-\infty}^{\infty} v^* (\, j w) Z(j\omega) v(j\omega) \; + \; \hat{u_T}^* (j\omega) G_2 \hat{u_T}(j\omega) \, d\omega$$

where

$$Z(j\omega) = [K_2XK_2 - K_2] + \omega^2[D_2/c^2 + M_2 - M_2XK_2 - K_2XM_2 + D_2XD_2] + w^4 M_2XM_2 \; .$$

Choose c^2 small enough that (1) $c^2M < D$ and (2) the coefficient of ω^2 is positive definite. If $XK_2 \geq I$, then Z is positive, and the controller is dissipative with supply rate w_c for all $E_2 \geq 0$. Therefore, the closed loop is stable (Theorem 2.3). That is, the closed loop system is stable for all E which satisfy

$$QE \geq F_2'K_2{}^{-1}F_2 \; .\square \tag{11}$$

Does this design have any physical meaning? The answer is positive. Consider the special case where the controller and the system have the same number of states, and furthermore assume that all states are measurable and the actuators are collocated with the sensors.

$$M\ddot{x}_1(t) + D\dot{x}_1(t) + Kx_1(t) = u_1 = r_1(t) - y_2(t) \tag{12}$$

$$y_1 = x_1.$$

Adjust the controller gain so $F_2 = K_2$:

$$M_2\ddot{x_2}(t) + D_2\dot{x_2}(t) + K_2x_2(t) = K_2u_2 = K_2(r_2(t) + y_1(t)) \tag{13}$$

$$y_2 = -K_2x_2 + Eu_2 = K_2x_2 + E(y_1 + r_2)$$

Defining $\underline{x} := (x_1, x_2)$ we can write the closed loop system as

$$\begin{bmatrix} M & 0 \\ 0 & M_2 \end{bmatrix} \ddot{\underline{x}}(t) + \begin{bmatrix} D & 0 \\ 0 & D_2 \end{bmatrix} \dot{\underline{x}}(t) + \begin{bmatrix} K+E & -K_2 \\ K_2 & K_2 \end{bmatrix} \underline{x}(t) = \begin{bmatrix} I & -E \\ 0 & K_2 \end{bmatrix} \underline{r}(t).$$

This is a stable second order system as long as the matrix

$$K_t = \begin{bmatrix} K+E & -K_2 \\ K_2 & K_2 \end{bmatrix}$$

is positive definite *i.e.* it represents the stiffness matrix for two "springs" K and K_2 connected in series. This is true if $E \geq K_2$. Noting that $F_2 = K_2$ and $Q = I$, this inequality is identical to the condition (11) arrived at using the dissipativeness criterion.

If we have scalar systems, the choice $E = k_2$ leads to a stable closed loop system. Setting the reference inputs to zero,

$$u_1 = -y_2 = k_2(x_2 - x_1)$$

The controller in this case reduces to a spring-mass-dashpot system connected in series with the system mass as shown in Figure 3.

For $d_2 = 0$, the system is just two spring-masses connected in series. As k_2 is increased, the stiffness of the system increases. For $d_2 > 0$, the system is always stable for $k_2 > 0$, and increased amounts of energy are dissipated in the dashpot as d_2 is increased. By adjusting the control system parameters, k_2, d_2, m_2 , we can design the closed system to be under-, critically or over-damped.

6 Conclusions

We have used results in dissipative system theory and Lynpunov's Second Method to develop stability theorems which generalize the passivity and small-gain theorems. Simple conditions for robust, model-independent controllers which stabilize non-passive structures were derived. Throughout the paper we have used the second-order form of the dynamical equations. Only displacement and velocity feedback were studied. However, the extension to accelerometers, and to combinations of these three types of sensors is straightforward.

Although for displacement feedback, a lower bound on the system damping is required, the controller is otherwise independent of the system parameters. It should be emphasized that this is a robustness result with respect to structural uncertainty in the absence of measurement uncertainty and other contributing factors.

Control performance is, of course, dependent on the system characteristics. Knowledge of the system model can always help improve a controller design. Future work will explore combining satisfaction of the appropriate dissipative condition with design techniques such as LQG regulator and H_∞ methods. Finally, the controller has been formulated from the continuous-time setting. Actual implementation of the controller, however, most likely requires usage of a digital computer. In future work, effects of sampling and time delays will be addressed. Other practical issue that can also affect the control performance such as measurement noises, and actuator and sensor saturation limits will be investigated.

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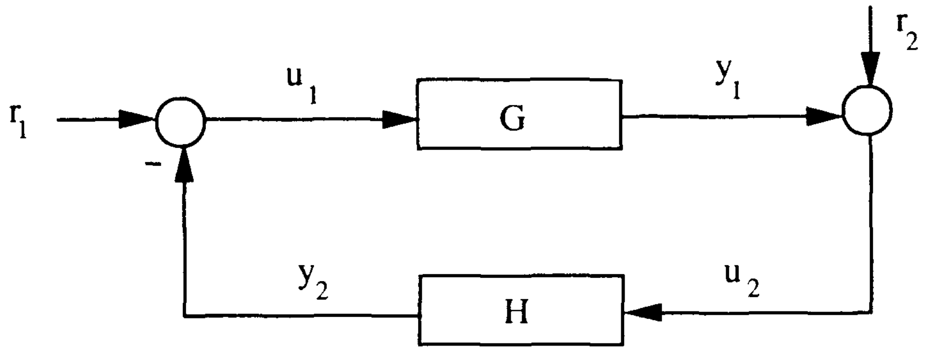

Figure 1: Feedback System

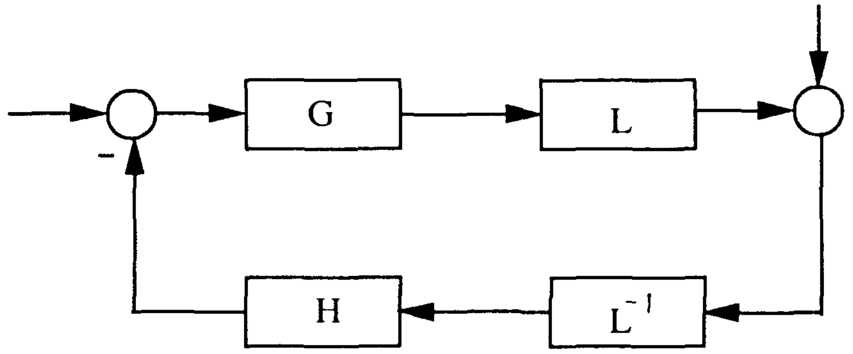

Figure 2: Feedback System with Multipliers

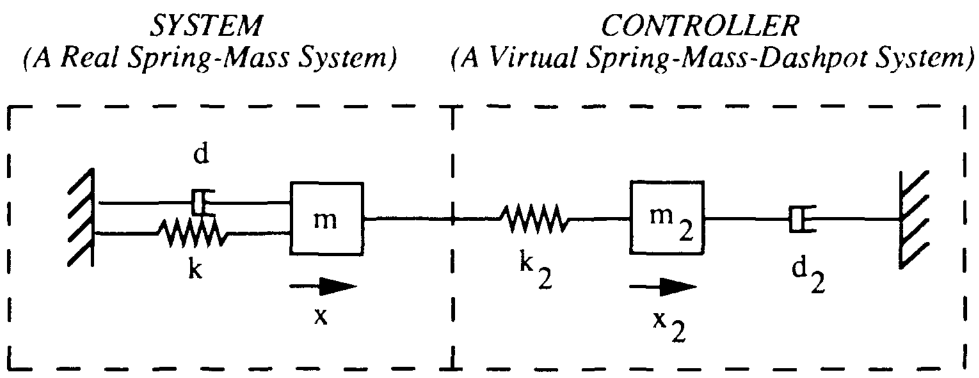

Figure 3: A simple spring-mass system with a simple spring-mass controller

NASA National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-187452 ICASE Report No. 90-65		

4. Title and Subtitle
DISSIPATIVE CONTROLLER DESIGNS FOR SECOND-ORDER DYNAMIC SYSTEMS

5. Report Date
September 1990

6. Performing Organization Code

7. Author(s)
K. A. Morris
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8. Performing Organization Report No.
90-65

10. Work Unit No.
505-90-21-01

9. Performing Organization Name and Address
Institute for Computer Applications in Science and Engineering
Mail Stop 132C, NASA Langley Research Center
Hampton, VA 23665-5225

11. Contract or Grant No.
NAS1-18605

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665-5225

13. Type of Report and Period Covered
Contractor Report

14. Sponsoring Agency Code

15. Supplementary Notes
Langley Technical Monitor:
Richard W. Barnwell

Final Report

16. Abstract

The passivity theorem may be used to design robust controllers for structures with positive transfer functions. This paper extends this result to more general configurations using dissipative system theory. A stability theorem for robust, model-independent controllers of structures which lack collocated rate sensors and actuators is given. The theory is illustrated for non-square systems with displacement sensors.

17. Key Words (Suggested by Author(s))
Robust Controller Design, Feedback Control Theory, dissipative systems, structural control

18. Distribution Statement
63 - Cybernetics

Unclassified - Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	16	A03